STABILITY OF PNEUMATIC and HYDRAULIC VALVES

These three tutorials will not be found in any examination syllabus. They have been added to the web site for engineers seeking knowledge on why valve elements sometimes go unstable and what can be done to prevent it.

TUTORIAL 3 - STABILITY ANALYSIS OF VALVES

This tutorial extends the work of the earlier tutorials to examine the causes of valve oscillations in hydraulic and pneumatic systems and includes some case studies.

The analogous quantities throughout will be as follows.

- Pressure (p) - Voltage (V)
- Mass flow (m) - Current (I or i)
- Mass (m) - Charge (Q)

TYPICAL CASE

The following case prompted the research outlined here but the general principles may be extended to a range of valves and situations. A simple spring loaded pressure limiting valve with a flat plate design was used to vent air from a vessel and was connected to the vessel by a pipe. It was found that the valve would go into violent oscillations at frequencies in excess of 50 Hz.

There are three forces acting on the valve plate: the force of the escaping air \( F_v \), the force of the steel spring \( F_s \) and the pneumatic damping force due to the pressure of the air in the dashpot \( F_d \). Each of these forces varies with distance \( x \). These interact with the valve mass and the pipeline dynamics to produce instability. Perturbations in the pressure travel along the pipe at the sonic velocity and are reflected from the volume as rarefactions (negative pressure). If the pipe length is resonant at a frequency near to the resonant frequency of the valve, they will interact. (Note that at closed ends such as a hydraulic pump, the pressure perturbations are reflected as positive pressure).

Research revealed that on part of the operating characteristic the mass flow could increase with the inlet pressure decreasing. This gives the device a negative characteristic or restriction. A similar electronic device is a tunnel diode used to create microwave oscillations.

Research also revealed that the pneumatic dashpot behaved like a spring and produced negligible damping until a damping orifice was added to it. Also the steel spring, having a moving mass, produces a force that gets out of phase with movement and decreases with frequency so that it became ineffective. Much of the work uncovered has been explained in tutorials (1) and (2). The following gives some insight into the stability analysis.
**DYNAMIC STABILITY**

Valve oscillation has several causes. The main ones are the interaction between the valve mass and spring dynamics and the dynamics of the pipe system connected to it.

Water hammer is a phenomenon involving pressure fluctuations moving up and down pipes, usually due to the closure or opening of a pipe line valve. You will find information on this in the fluid mechanics tutorials. A related phenomenon occurs in hydraulic and pneumatic systems with interaction between the valve and the pipe dynamics. When a valve moves, the flow rate through it changes and the pressure in the pipe changes. Pressure changes can travel along a pipe and are reflected from closed end as a pressure and from open ends as rarefaction. It becomes possible for the pressure and flow rate to get out of phase with each other and with the valve position and thus sustain the oscillation of the valve element.

An additional cause is that the possibility of a negative pressure-flow rate characteristic, a well known cause of instability.

**CLOSED LOOP MODEL**

Two models are shown. One is based on the relationship between flow rate and valve position and the other is based on the relationship between valve movement and the fluid force acting on the valve element. If a transfer function is derived for the two blocks and a closed loop transfer function created, then a stability analysis can be made. This model may be applied to all manner of valves, hydraulic and pneumatic. The dynamics of the pipe line may be determined in many cases by applying electrical transmission line theory to it.

Generally, when the resonant frequency of the valve is close to the quarter wave resonant frequency of the pipe (or some multiple of the frequency) the valve will be unstable. There will be a 90° phase shift between the flow and the pressure (like between current and voltage in an electrical system).

In the following a great simplification is used where the changes in the variables is assumed to be very small and so the relationships between them are linearised to the gradients of the functions at the operating point. This is called a ‘Small Perturbation Analysis’ and allows some insight to the onset of instability. Larger perturbations may make the system more or less stable.
ANALYSIS OF THE VALVE DYNAMICS

When a valve oscillates, the whole system becomes dynamic and surprising things can happen like what happens to an ordinary spring which we will examine first.

DYNAMIC SPRING RATE

An ordinary spring has a spring rate \( k_s = \Delta F/\Delta x \) but the spring coils have a mass and only one end moves so some of the mass moves faster than the rest. When one end is moved harmonically the actual spring rate or dynamic spring rate has been shown to be

\[
k_{sd} = k_s \omega \sqrt{\frac{M_s}{k_s}} \tan\left( \omega \sqrt{\frac{M_s}{k_s}} \right)
\]

In a later case study \( m_s = 1.46 \text{ g} \) and \( k_s = 1.46 \text{ N/mm} \) and the plot is shown. This shows that the spring rate can become zero or even negative if it is attached to the valve element.

SMALL PERTURBATION METHOD

In this section the analysis is made by assuming that all the changes are small. This is called a small perturbation analysis.

We need to consider how the mass flow rate \( m \) and force \( F_v \) varies with pressure \( p_1 \) and opening \( x \). \( F_v \) is the fluid force acting on the valve element due to the pressure and momentum change of the fluid.

These are related by some function. At the operating point the gradients of the functions are \( C_1 \), \( C_2 \), \( C_3 \) and \( C_4 \). The graphs are only for illustrative purposes.

For a small change we consider that the relationship is linear. Clearly if the changes are large the result is different but this method is useful for determining the likelihood of instability occurring.

VALVE IMPEDANCE and TRANSFER FUNCTIONS

Many models use impedance to model the dynamics of the valve and this fits in with the electrical analogue. The following could apply to most types of hydraulic and pneumatic valves but a simple flat valve will be considered here. The mass flow rate \( \dot{m} \) depends on the opening \( x_o \) and the pressure drop over the element \( \Delta p = p_1 - p_2 \).

We will consider \( p_2 \) as constant so \( \delta \Delta p = \delta p_1 \).
IMPEDANCE

The impedance is defined as \( Z_v = \frac{\delta p_1}{\delta \dot{m}} \)

The change in flow rate will be partly due to the change in opening and partly due to the change in pressure so \( \delta \dot{m} = C_1 \delta p_1 + C_2 \delta x \)

\[
C_1 = \frac{\delta \dot{m}}{\delta p_1}, \quad C_2 = \frac{\delta \dot{m}}{\delta x}, \quad \delta p_1 = \left( C_1 + C_2 \frac{\delta x}{\delta p_1} \right)^{-1} \quad \text{……………(1)}
\]

This shows that the impedance is negative when \( C_2 \frac{\delta x}{\delta p_1} > C_1 \)

The change in the force \( F_v \) will be partly due to the change in opening and partly due to the change in pressure so \( \delta F_v = C_3 \delta p_1 + C_4 \delta x \) \quad \text{……………(2)}

The force '\( F_v \)' is opposed by various forces depending on the design. This could include the force due to the steel spring '\( F_s \)', the damping force '\( F_d \)' due to the dashpot and the inertia force '\( F_i \)' due to the mass of the element '\( M \)'.

Equating the forces we have \( F_v = F_s + F_d + F_i \)

If these change slightly then \( \delta F_v = \delta F_s + \delta F_d + \delta F_i \)

The inertia force is defined as \( F_i = M \frac{d^2 x}{dt^2} \) where M is the moving mass.

Each of these forces varies with opening so we define the gradients of the functions as a spring rate such that: \( k_d = \frac{\delta F_d}{\delta x}, \quad k_s = \frac{\delta F_s}{\delta x}, \quad k_v = \frac{\delta F_v}{\delta x} \)

\[
\delta F_v = k_s \delta x + k_d \delta x + M \frac{d^2(\delta x)}{dt^2} = \delta x(k_s + k_d) + M \frac{d^2(\delta x)}{dt^2}
\]

In Laplace form this is \( \delta F_v = \delta x \left( k_s + k_d + M s^2 \right) \)

\[
\delta F_v = \delta x \left( k_s + k_d + M s^2 \right) = \delta x \left( k_s + k_d + M s^2 \right) \quad \text{……………(3)}
\]

Equate (3) and (2) \( C_3 \delta p_1 + C_4 \delta x = \delta x \left( k_s + k_d + M s^2 \right) \)

\[
C_3 \delta p_1 = \delta x \left( k_s + k_d + M s^2 \right) - C_4 \delta x = \delta x \left( k_s + k_d + M s^2 - C_4 \right)
\]

\[
\frac{C_3}{k_s + k_d + M s^2 - C_4} = \frac{\delta x}{\delta p_1} \text{ Substitute into (1) } Z_v = \left( C_1 + \frac{C_2 C_3}{k_s + k_d + M s^2 - C_4} \right)^{-1}
\]

If the valve oscillates harmonically with a small amplitude we may substitute \( s = j \omega \)

\[
Z_v = \left( C_1 + \frac{C_2 C_3}{k_s + k_d - C_4 - M \omega^2} \right)^{-1} \text{ this is negative when } \left[ \frac{\omega}{\frac{C_2 C_3}{C_1} + \frac{k_s + k_d - C_4}{M}} \right]^{1/2}
\]

The resonant frequency is \( \omega_r = \frac{1}{M} \left( k_s + k_d - \frac{\delta F_v}{\delta x} \right) \)

As discussed above, the steel spring rate a function of frequency and the dynamic rate \( k_{sd} \) should be used. This may give a negative value at the frequency of interest and can greatly increase the probability of negative valve impedance.

Negative impedance will occur when the net spring rate is negative.
Many valves have a characteristic that makes the restriction to flow decrease with movement such as poppet valves and spool valves. Variable area flow meters (Rotameters) are based on this principle and the floats can often oscillate up and down inside the tube due to this. Electronic devices such as tunnel diodes have a similar characteristic and are used to generate electronic oscillations.

ANALYSIS OF THE PIPELINE DYNAMICS

Consider a pipe connected to volume as shown. The pipe has a length \( l \) and bore area \( A_1 \). The fluid in the pipe has inertance \( L \), capacitance \( C \) and resistance \( G_1 \). The analogue circuit is shown. \( p_v \) is equivalent to \( V_i \) and \( p_1 \) is equivalent to \( V_o \).

$$\begin{align*}
\text{The circuit impedance viewed from the volume is:} & \quad Z_p = G_1 + XL + X_c \\
\text{This is the LUMPED PARAMETERS. It was shown in the first tutorial that for a gas} & \quad C = \frac{A\ell}{a^2} \text{ and} \\
L &= \frac{\ell}{A} \text{. The resistance is less easy to express.} \\
\text{Well established electrical transmission line theory lets us make the analogy that the volume is the} & \quad \text{same as a short circuit in which case the impedance in terms of the DISTRIBUTED} \\
\text{PARAMETERS is} & \quad Z_p = \frac{\Delta p_1}{\Delta m} = -jZ_0 \tan \left( \frac{\omega \ell}{a} \right)
\end{align*}$$

Consider that a simple valve is placed at the end of the pipe. In this case we will assume that the restriction of the valve at the operating point is \( G_2 \) but of course this could be a complex expression.

**METHOD 1 LUMPED PARAMETERS CIRCUIT**

$$\begin{align*}
\text{The circuit impedance viewed from the volume is:} & \quad Z = G_1 + XL - \frac{G_2}{X_c/(X_c - G_2)} \\
X_L \text{ is the inductive reactance and in Laplace form} & \quad X_L = sL \\
X_C \text{ is the capacitive reactance and in Laplace form} & \quad X_C = 1/sC \\
Z = G_1 + sL - \frac{G_2/sC}{(1/sC) - G_2} & = G_1 + sL - \frac{G_2}{1 - G_2 sC} = G_2 - G_1 + s(G_1 G_2 C - L) + s^2 G_2 LC \\
\text{The numerator is} & \quad s^2 G_2 LC + s(G_1 G_2 C - L) + G_2 - G_1
\end{align*}$$
Apply the Routh-Hurwitz criteria and the requirements for stable operation are:

i. \( G_1 > G_2 \)

ii. \( G_1 > \frac{L}{G_2 C} \)

For a pneumatic system the second criteria reduces to \( G_1 > \frac{a^2}{A^2 G_2} \)

The stabilising factors are a large capacitance and a suitable pipe restriction that depends on the friction coefficient of the pipe and the length.

In many instances, there will be an operating point of the valve where the mass flow rate increases even when the pressure \( p_1 \) reduces and this means there is an operating point with a negative \( G_2 \) and such systems are unstable.

**METHOD 2 CLOSED LOOP with DISTRIBUTED PARAMETERS**

The volume at the supply end of the pipe is analogous to a closed circuit in electrical theory and well established transmission line theory gives the impedance of the line as viewed from that end. Making a pneumatic analogy we have the pipe impedance viewed from the volume as:

\[
Z_p = \frac{\Delta p_1}{\Delta m} = -jZ_o \tan \left( \frac{\omega l}{a} \right)
\]

\( Z_o \) is characteristic impedance of the pipe and by analogy is \( Z_o = \sqrt{\frac{L}{C}} = \frac{a}{A} \)

\( L \) and \( C \) are the distributed parameters with negligible restriction (friction).

We had \( \frac{\delta p_1}{\delta m} = \left( C_1 + C_2 \frac{\delta x}{\delta p_1} \right)^{-1} \) …..(1) \( \delta F_v = \delta x (k_s + k_d + M s^2) \) …..(3)

Combine equations (1), (3) and \( Z_p = \frac{\Delta p_1}{\Delta m} \) to eliminate \( \delta p_1 \) and we get the result

\[
\frac{\Delta F_v}{\Delta x} = \frac{C_3 C_2 Z_p}{1 - Z_p C_1} + C_4
\]

This is a transfer function \( H_2 \) relating the fluid forces on the valve element to the pressure in the pipe. \( H_2 = \frac{C_3 C_2 Z_p}{1 - Z_p C_1} + C_4 \)

It was shown earlier that the relationship between the force and the pressure from examination of the pipeline is

\[ H_1 = \left( k_s + k_d - M \omega^2 \right)^{-1} \]

The closed loop block diagram is shown. \( H_1 \) is the transfer function of the valve and \( H_2 \) the transfer function of the pipe line.

The open loop transfer function is:

\[ H_{OL} = H_1 H_2 = \left( k_s + k_d - M \omega^2 \right)^{-1} \left( \frac{C_3 C_2 Z_p}{1 - Z_p C_1} + C_4 \right) \]

Remember in terms of distributed parameters that \( Z_p = -jZ_o \tan \left( \frac{\omega l}{a} \right) \)

When \( \omega l/a = \pi/2, 3\pi/2, \ldots \) this is infinity and when \( \omega l/a = \pi, 2\pi, 3\pi, \ldots \) it is zero.
If we could plot the Nyquist diagram the system would be unstable if it enclosed the -1 point. Since
the constants \( C_3 \) and \( C_4 \) are difficult to determine this might be difficult to do but we do know that
this would occur if \( H_1 \) or \( H_2 \) is negative and when:

i. When \( \omega \theta/a = \pi/2, 3\pi/2 \ldots \) \( \omega > \sqrt{\frac{k_s + k_d - C_4 + C_3C_2/C_1}{M}} \)

ii. When \( \omega \theta/a = \pi, 2\pi, 3\pi \ldots \) \( \omega > \sqrt{\frac{k_s + k_d - C_4}{M}} \)

This is a logical result.

Condition (i) corresponds to the pipe being resonant due to being ¼ wave length long or a multiple
of this. At this condition \( C_3C_2/C_1 \) changes sign but \( C_4 \) being independent of pressure change will
remains negative.

Condition (ii) corresponds to the pipe being resonant and ½ wave length long. At this condition the
pressure variations in the pipe being reflected will cancel the variations being set up by the valve
oscillation. The component \( C_3C_2/C_1 \) vanishes but \( C_4 \) will remains negative.

If anyone has further data on this topic and would like to add it to the tutorial, please
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