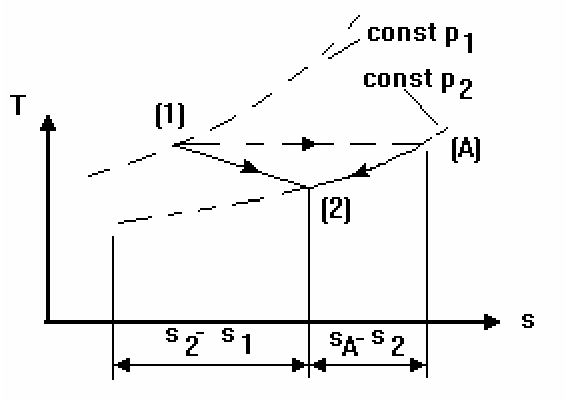
Q3 (a) Show that for a perfect gas undergoing a process the entropy change is given by :

$$S_2 - S_1 = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR\ln\left(\frac{p_2}{p_1}\right)$$

(b) Air flows steadily and adiabatically through a duct of varying area. At section X the pressure and temperature are 2.5 bar and 40°C respectively. A measuring instrument at that section indicates a velocity of 222 m/s without indicating the direction. At another section Y the and temperature are 4.5 bar and 60°C respectively.

(i) Calculate the velocity at Y.
(ii) Calculate the specific entropy change between the two sections and deduce the direction of flow.

(a) DERIVATION




The polytropic expansion is from (1) to (2) on the T-s diagram with different pressures, volumes and temperatures at the two points. The derivation is done in two stages by supposing the change takes place first at constant temperature from (1) to (A) and then at constant pressure from (A) to (2). You could use a constant volume process instead of constant pressure if you wish.

$s_2$-$s_1$ = ($s_A$-$s_1$) - ($s_A$-$s_2$)
$s_2$-$s_1$ = ($s_A$-$s_1$) + ($s_2$-$s_A$)

For the constant temperature process
($s_A$-$s_1$) = R ln($p1/p_A$)

For the constant pressure process
($s_2$-$s_A$) = ($c_p$) ln($T_2/T_A$)

Hence

$$\Delta s = R\ln\frac{p_1}{p_A} + C_p \ln\frac{T_2}{T_A} + s2\text{-}s1$$ Since $p_A = p_2$ and $T_A = T_1$

Then $$S_2 - S_1 = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR\ln\left(\frac{p_2}{p_1}\right)$$

(b) (i)Apply the steady flow energy equation between X and Y

$$T_x + \frac{u_x^2}{2c_p} = T_y + \frac{u_y^2}{2c_p} \qquad 313 + \frac{222^2}{2c_p} = 333 + \frac{u_y^2}{2c_p} \qquad 313 - 333 + \frac{222^2}{2c_p} = \frac{u_y^2}{2c_p}$$

$$-20 + \frac{222^2}{2c_p} = \frac{u_y^2}{2c_p} \qquad -40c_p + 222^2 = u_y^2$$

The mean temperature is 323K and from the tables $c_p$ = 1.0051 kJ/kg K
$u_y = \sqrt{}(222^2 - 40 \times 1005.1)$ = 95.29 m/s

(ii) $$s_2 - s_1 = c_p \ln\left(\frac{T_y}{T_x}\right) - R\ln\left(\frac{p_y}{p_x}\right)$$ R = 287.1 J/kg K

$$s_2 - s_1 = 1005.1\ln\left(\frac{333}{313}\right) - 287.1\ln\left(\frac{4.5}{2.5}\right) = -106.5 \text{ J/kgK}$$ Since this is negative, the flow must be the other way from Y to X.