SOLUTIONS C106 THERMODYNAMIC, FLUID AND PROCESS ENGINEERING
Year 2004

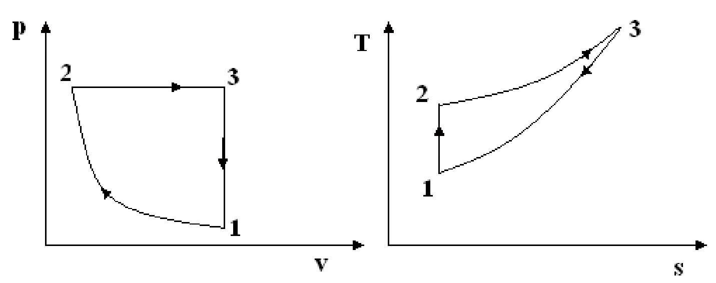
Q1 A heat engine cycle using a perfect gas as the working fluid consists of the following three non-flow reversible processes.

1 – 2 adiabatic compression.
2 – 3 constant pressure heating.
3 – 1 constant volume cooling.

(a) sketch the p = v and T – s diagrams.

(b) show that the cycle efficiency is given by

$$\eta = 1 - \frac{1-\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}}{\gamma\left[1-\left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}\right]}$$



The thermal efficiency of any cycle is given by $\eta = 1 - \frac{Q_{out}}{Q_{in}}$

The heat input is 2-3 $Q_{in} = mc_p(T_3\text{-}T_2)$
The heat output is 3-1 $Q_{in} = mc_v(T_3\text{-}T_1)$

$$\eta = 1 - \frac{mc_v(T_3 - T_1)}{mc_p(T_3 - T_2)} = 1 - \frac{c_v(T_3 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{(T_3 - T_1)}{\gamma(T_3 - T_2)}$$ noting $c_p/c_v = \gamma$ $$\eta = 1 - \frac{\left(1 - \frac{T_1}{T_3}\right)}{\gamma\left(1 - \frac{T_2}{T_3}\right)}$$

Combining $pv^\gamma = c$ and $pv/T = C$ we can show that $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$

since $v_1 = v_3$ and $p_3 = p_2$ then $\frac{T_2}{T_1} = \left(\frac{v_3}{v_2}\right)^{\gamma-1}$ $\quad \frac{v_2}{v_3} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$ and $\frac{p_1}{p_3} = \left(\frac{T_1}{T_3}\right)^{\frac{\gamma}{\gamma-1}}$

From $\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$ $\qquad \frac{T_1}{T_3} = \frac{p_1 v_1}{p_3 v_3} = \frac{p_1}{p_3} = \left(\frac{T_1}{T_3}\right)^{\frac{\gamma}{\gamma-1}}$ $\quad \frac{T_2}{T_3} = \frac{p_2 v_2}{p_3 v_3} = \frac{v_2}{v_3} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$

$$\eta = 1 - \frac{\left(1 - \frac{T_1}{T_3}\right)}{\gamma\left(1 - \frac{T_2}{T_3}\right)} = 1 - \frac{1-\left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}}{\gamma\left[1-\left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}\right]}$$