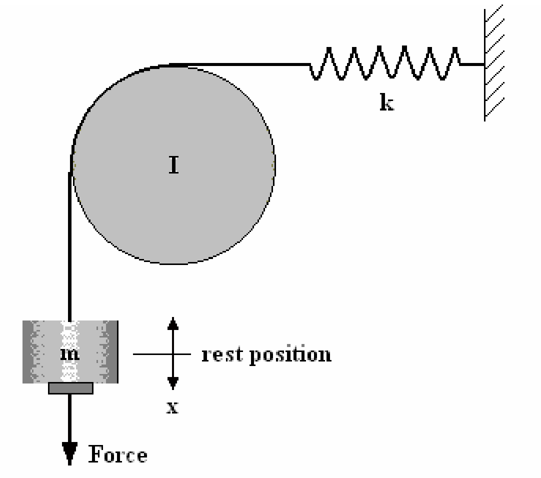
MECHANICAL AND STRUCTURAL ENGINEERING C105
EXAM QUESTIONS 2005 Q6

A rope which is connected to a spring at one end and carries a mass of 8 kg at the other end, passes over a pulley as shown.



The spring has a stiffness of 12 N/mm. The pulley has a mass of 6 kg and an effective diameter of 300 mm. Consider the pulley to be a uniform disc for which the the moment of inertia is $mr^2/2$.Assume the rope has negligible mass and does not slip.

Determine the natural frequency of vibration.

SOLUTION

The solution must be done analytically since it is a combination of torsional and linear oscillations. Suppose the mass is pulled downwards with a force F. This must overcome the inertia of the mass, the inertia of the drum and stretch the spring.

Inertia force to accelerate the drum

The Torque required to overcome the inertia of the drum is $T = I\alpha$

Torque = Force x radius or T= F R and the force is F =T/R

Substitute $T = I\alpha$

$\mathbf{F_{i1} = I\alpha/R}$ where α is the angular acceleration of the drum.

Inertia force to accelerate the mass

$\mathbf{F_{i2} = m\,a}$ where a is the linear acceleration.

Force to stretch the spring

$\mathbf{F_s = k\,x}$ where k is the spring stiffness.

Force balance $F = F_{i1} + F_{i2} + F_s = I\alpha/R + ma + kx$

The angular acceleration is linked to the linear acceleration by $\alpha = a/R$ where R is the drum radius.

$$F = Ia/R^2 + ma + kx = a(m + I/r^2) + kx$$

For a free oscillation F = 0 hence

$$0 = \frac{Ia}{R^2} + ma + kx = a\left(\frac{I}{R^2} + m\right) + kx \qquad \text{Make a the subject} \quad a = -\left(\frac{k}{\frac{I}{R^2} + m}\right)x$$

This shows that the acceleration is directly proportional to displacement so the motion must be simple harmonic. The constant of proportionality is the angular frequency squared so:

$$\omega^2 = \left(\frac{k}{\frac{I}{R^2} + m}\right) \qquad \omega = \sqrt{\left(\frac{k}{\frac{I}{R^2} + m}\right)} \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{k}{\frac{I}{R^2} + m}\right)}$$

Evaluate f using k = 12 000 N/m, m = 8 kg , r = 0.15 m and $I = mr^2/2 = 6 \times 0.15^2/2 = 0.0675$ kg m^2

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{12\,000}{\frac{0.0675}{0.15^2} + 8}\right)} = 5.26\text{Hz}$$