

MECHANICAL AND STRUCTURAL ENGINEERING C105
EXAM QUESTIONS 2005 Q5

Two vehicles A and B are involved in a head-on collision on a motorway where the speed limit is 120 km/h. Vehicle A has a total mass of 900 kg and vehicle b has a total mass of 1300 kg.

Accident investigators made the following observations at the scene of the accident.

Vehicle A started to skid 43 m before the impact and then rebounded 11 m from the point of impact. Vehicle B started to skid 37 m before the point of impact and then rebounded 6 m from the point of impact.

Road conditions at the time of the accident indicated a coefficient of friction between road and tyre for both vehicles was 0.8.

Structural assessment of the two vehicles led to an estimate of the coefficient of restitution between the two vehicles of 0.4.

Based on the information provided, determine

- a) The speeds of the two vehicles at the instant they collided.
- b) Whether or not the vehicles were travelling in excess of the speed limit at the point where they started to skid.

It appears that the following method is used to solve this where it is assumed that when they rebound they skid back the road.

VEHICLE A

$$\text{Kinetic energy at impact} = mv^2/2 = 900 v^2/2 = 450 v^2$$

$$\text{Energy lost by skidding} = \text{Force} \times \text{distance moved}$$

$$\text{Force} = \mu W = \mu mg$$

$$\text{Energy Lost} = 0.8 \times 900 \times 9.81 \times 11 = 77695 \text{ J}$$

$$v^2 = 77695/450 = 172.6 \quad v = 13.14 \text{ m/s}$$

VEHICLE B

$$\text{Kinetic energy at impact} = mv^2/2 = 1300 v^2/2 = 650 v^2$$

$$\text{Energy lost by skidding} = \text{Force} \times \text{distance moved}$$

$$\text{Force} = \mu W = \mu mg$$

$$\text{Energy Lost} = 0.8 \times 1300 \times 9.81 \times 6 = 61214 \text{ J}$$

$$v^2 = 61214/650 = 94.2 \quad v = 9.7 \text{ m/s}$$

Now use the conservation of momentum and coefficient of restitution.

$$\text{Relative velocity after impact} = 13.14 + 9.7 = 22.84 \text{ m/s (separating)}$$

$$\text{Relative velocity before impact} = e \times 22.84 = 0.4 \times 22.84 = 9.136 \text{ m/s (coming together)}$$

$$v_A = 9.136 - v_B$$

Total Momentum before impact

$$M_A v_A + M_B v_B = 900 v_A + 1300 v_B$$

Total Momentum after impact

$$M_A u_A + M_B u_B = 900 \times 13.136 - 1300 \times 9.7 = -787.6$$

$$900 v_A + 1300 v_B = -787.6$$

$$900 (9.136 - v_B) + 1300 v_B = -787.6$$

$$8222.4 - 900 v_B + 1300 v_B = -787.6$$

$$400 v_B = -9020$$

$$v_B = -22.55 \text{ m/s} \quad v_A = 9.136 - (-22.55) = 31.7 \text{ m/s}$$

Now calculate the original velocities from the skidding as they approached.

VEHICLE A

$$\text{Kinetic energy change} = m(v^2 - 31.7^2)/2 = 900(v^2 - 31.7^2)/2 = 450(v^2 - 31.7^2)$$

$$\text{Energy lost due to skidding} = 0.8 \times 900 \times 9.81 \times 43 = 303718 \text{ J}$$

$$\text{Equate } 450(v^2 - 31.7^2) = 303718 \text{ J} \quad v^2 = 16799 \quad v = 41 \text{ m/s for vehicle A}$$

Original velocity was 147.6 km/h so it was exceeding the speed limit.

VEHICLE B

$$\text{Kinetic energy change} = m(v^2 - 22.55^2)/2 = 1300(v^2 - 22.55^2)/2 = 650(v^2 - 22.55^2)$$

$$\text{Energy lost due to skidding} = 0.8 \times 1300 \times 9.81 \times 37 = 377489 \text{ J}$$

$$\text{Equate } 650(v^2 - 22.55^2) = 377489 \text{ J} \quad v^2 = 1089 \quad v = 33 \text{ m/s for vehicle B}$$

Original velocity was 118.8 km/h so it was not exceeding the speed limit.