ENGINEERING COUNCIL
CERTIFICATE LEVEL

ENGINEERING SCIENCE C103

TUTORIAL 5 - THIN WALLED VESSELS

You should judge your progress by completing the self assessment exercises. These may be sent for marking at a cost (see home page).

When you have completed this tutorial you should be able to do the following.

- Define a thin walled cylinder.
- Solve circumferential and longitudinal stresses in thin walled cylinders.
- Solve circumferential and longitudinal stresses in thin walled spheres.
- Calculate the bursting pressure of thin walled cylinders and spheres.
- Calculate changes in diameter and volume due to pressure.
- Solve problems involving the compression of fluids into pressure vessels.
- Solve problems involving interference fits between shafts and sleeves.

1.

1. THIN WALLED CYLINDER.

A cylinder is regarded as thin walled when the wall thickness t is less than 1/20 of the diameter D. When the wall is thicker than this, it is regarded as a thick wall and it is treated differently as described later.

Consider a cylinder of mean diameter D, wall thickness t and length L. When the pressure inside is larger than the pressure outside by p, the cylinder will tend to split along a length and along a circumference as shown in figures 1 and 2.

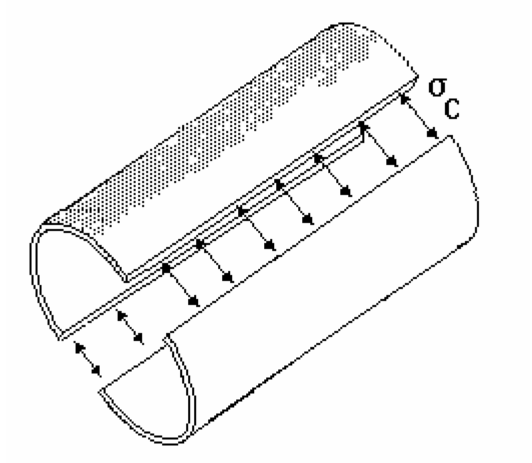

Figure 1

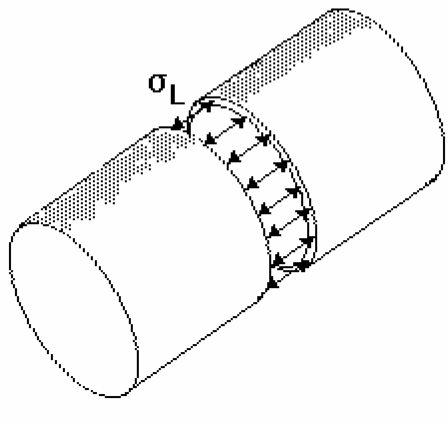

Figure 2

The stress produced in the longitudinal direction is $\sigma_L$and in the circumferential direction is σ_C. These are called the longitudinal and circumferential stresses respectively. The latter is also called the hoop stress.

Consider the forces trying to split the cylinder about a circumference (fig.2). So long as the wall thickness is small compared to the diameter then the force trying to split it due to the pressure is

$$F = pA = p\frac{\pi D^2}{4} \text{..........................(1.1)}$$

So long as the material holds then the force is balanced by the stress in the wall. The force due to the stress is

$$F = \sigma_L \text{ multiplied by the area of the metal} = \sigma_L \pi D t \text{ ....(1.2)}$$

Equating 1.1 and 1.2 we have

$$\sigma_L = \frac{pD}{4t} \text{ ....(1.3)}$$

Now consider the forces trying to split the cylinder along a length.
The force due to the pressure is

$$F = pA = pLD \text{ ....(1.4)}$$

So long as the material holds this is balanced by the stress in the material. The force due to the stress is

$$F = \sigma_C \text{ multiplied y the area of the metal} = \sigma_C 2Lt \text{ ....(1.5)}$$

Equating 1.4 and 1.5 we have

$$\sigma_C = \frac{pD}{2t} \text{ ....(1.6)}$$

It follows that for a given pressure the circumferential stress is twice the longitudinal stress.

<u>WORKED EXAMPLE No.1</u>

A cylinder is 300 mm mean diameter with a wall 2 mm thick. Calculate the maximum pressure difference allowed between the inside and outside if the stress in the wall must not exceed 150 MPa.

<u>SOLUTION</u>

The solution must be based on the circumferential stress since this is the largest.

σ_C = pD/2t = 150 MPa

p = 150 MPa x 2t/D = 150 x 2 x 0.002/0.3

p = 2 MPa

2. THIN WALLED SPHERE

A sphere will tend to split about a diameter as shown in fig.3

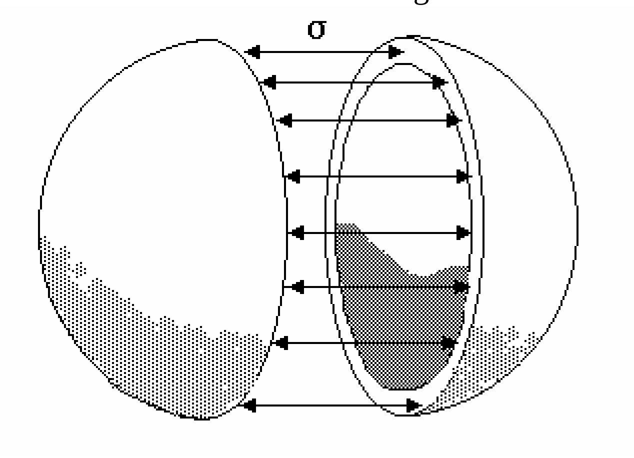

Figure 3

The stress produced in the material is equivalent to the longitudinal stress in the cylinder so

$$\sigma_C = \frac{pD}{4t} \quad \text{.................................(2.1)}$$

WORKED EXAMPLE No.2

Calculate the maximum allowable pressure difference between the inside and outside of a sphere 50 mm mean diameter with a wall 0.6 mm thick if the maximum allowable stress is 1.5 MPa.

SOLUTION

Using equation G we have
$\sigma = pD/4t = 150$ MPa
$p = 1.5\text{x}10^6 \text{ x } 4t/D = 1.5\text{x}10^6 \text{ x } 4 \text{ x } 0.0006/0.05 = 72$ kPa

SELF ASSESSMENT EXERCISE No.1

1. A thin walled cylinder is 80 mm mean diameter with a wall 1 mm thick. Calculate the longitudinal and circumferential stresses when the inside pressure is 500 kPa larger than on the outside. (Answers 10 MPa and 20 MPa).

2. Calculate the wall thickness required for a thin walled cylinder which must withstand a pressure difference of 1.5 MPa between the inside and outside. The mean diameter is 200 mm and the stress must not exceed 60 MPa. (Answer 2.5 mm)

3. Calculate the stress in a thin walled sphere 100 mm mean diameter with a wall 2 mm thick when the outside pressure is 2 MPa greater than the inside. (Answer -25 MPa).

3. VOLUME CHANGES

We will now look at how we calculate the changes in volume of thin walled vessels when they are pressurised.

CYLINDERS

Consider a small rectangular area which is part of the wall in a thin walled cylinder (figure 4).

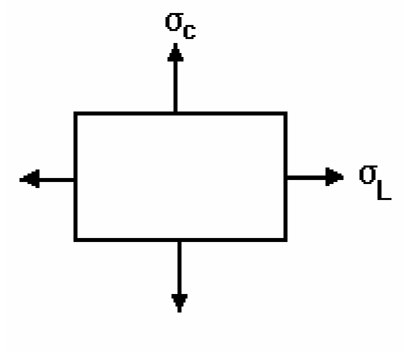

Figure 4

There are two direct stresses perpendicular to each other, σ_C and σ_L. From basic stress and strain theory (tutorial 1), the corresponding longitudinal strain is :

$$\varepsilon_L = \frac{1}{E}(\sigma_L - \nu\sigma_C)$$

E is the modulus of elasticity and ν is Poisson's ratio. Substituting σ_L= pD/4t and σ_C=pD/2t we have

$$\varepsilon_L = \frac{\Delta L}{L} = \frac{1}{E}\left(\frac{pD}{4t} - \nu\frac{pD}{2t}\right) = \frac{pD}{4tE}(1-2\nu).......................(3.1)$$

The circumferential strain may be defined as follows.

ε_C= change in circumference/original circumference

$$\varepsilon_C = \frac{\pi(D+\Delta D) - \pi D}{\pi D} = \frac{\Delta D}{D}$$

The circumferential strain is the same as the strain based on diameter, in other words the diametric strain.

From basic stress and strain theory, the corresponding circumferential strain is :

$$\varepsilon_C = \frac{1}{E}(\sigma_C - \nu\sigma_L)$$

Substituting σ_L= pD/4t and σ_C=pD/2t we have

$$\varepsilon_C = \varepsilon_D = \frac{\Delta D}{D} = \frac{1}{E}\left(\frac{pD}{2t} - \nu\frac{pD}{4t}\right) = \frac{pD}{4tE}(2-\nu).......................(3.2)$$

Now we may deduce the change in diameter, length and volume.

Original cross sectional area of cylinder = $A_1 = \pi D^2/4$

Original length = L_1 Original volume =$V_1 = A_1\ L_1 = (\pi D^2/4)(L_1)$

New cross sectional area = $A_2 = (\pi \Delta D + \Delta D)^2$

New length = $L_2 = L + \Delta L$

New volume = $V_2 = A_2 L_2 = \{(\pi\Delta D + \Delta D)^2\}(L_1 + \Delta L)$

Change in volume = $\Delta V = V_2 - V_1$

Volumetric strain = $\varepsilon_V = \Delta V/V_1$

$$\varepsilon_V = \frac{\left(\frac{\pi(D+\Delta D)^2}{4}\right)(L_1+\Delta L) - \left(\frac{\pi D^2}{4}\right)L_1}{\left(\frac{\pi D^2}{4}\right)L_1} = \frac{\left(\frac{(D+\Delta D)^2}{4}\right)(L_1+\Delta L) - \left(\frac{D^2}{4}\right)L_1}{\left(\frac{D^2}{4}\right)L_1}$$

Dividing out and clearing brackets and ignoring the product of two small terms, this reduces to

$$\varepsilon_V = \frac{\Delta L}{L_1} + 2\frac{\Delta D}{D} = \varepsilon_L + 2\varepsilon_D \text{.............................(3.3)}$$

If we substitute equation 3.1 and 3.2 into this we find

$$\varepsilon_V = \frac{pD}{4tE}(5-4\nu) \text{.................................(3.4)}$$

WORKED EXAMPLE No.3

A cylinder is 150 mm mean diameter and 750 mm long with a wall 2 mm thick. It has an internal pressure 0.8 MPa greater than the outside pressure. Calculate the following.

i. The circumferential strain.
ii. The longitudinal strain.
iii. The change in cross sectional area.
iv. The change in length.
iv. The change in volume.

Take E = 200 GPa and ν = 0.25

SOLUTION

σ_C= pD/2t = 30 MPa σ_L= pD/4t = 15 MPa

ε_D= ΔD/D = (pD/4tE)(2 - ν) = 131.25 $\mu\varepsilon$

ΔD = 150 x 131.25 x 10^{-6} = 0.0196 mm D_2 = 150.0196 mm

A_1 = π x 150^2/4 = 17671.1 mm^2 A_2 = π x 150.0196^2/4 = 17676.1 mm^2

Change in area = 4.618 mm^2

ε_L= $\Delta L/L_1$ = (pD/4tE)(1 - 2ν) = 37.5$\mu\varepsilon$
ΔL= 750 x 37.5 x 10^{-6} = 0.0281 mm

Original volume = A_1L_1 = 13 253 600 mm^3
Final volume = A_2L_2 = 13 257 600 mm^3
Change in volume = 4000 mm^3
Check the last answer from equation 3.4
ε_V =(pD/4tE)(5 - 4ν) = 300 x 10^{-6}
Change in volume = V_1 x ε_V = 13 253 600 x 300 x 10^{-6} = 4000 mm^3

SPHERES

Consider a small rectangular section of the wall of a thin walled sphere. There are two stresses mutually perpendicular similar to fig. 4 but in this case the circumferential stress is the same as the longitudinal stress. The longitudinal strain is the same as the circumferential strain so equation 3.3 becomes

$\varepsilon_V = \varepsilon_D + 2\varepsilon_D$

$\varepsilon_V = 3\varepsilon_D$ ....(3.5)

The strain in any direction resulting from the two mutually perpendicular equal stresses is

$\varepsilon_D = (\sigma/E)(1-\nu)$

Hence $\varepsilon_V = 3(\sigma/E)(1-\nu)$ ....(3.6)

WORKED EXAMPLE No. 4

A sphere is 120 mm mean diameter with a wall 1 mm thick. The pressure outside is 1 MPa more than the pressure inside. Calculate the change in volume.

Take E = 205 GPa and ν= 0.26

SOLUTION

$\varepsilon_V = 3(\sigma/E)(1-\nu) = -324.87\mu\varepsilon$

(note the sphere shrinks hence the negative sign)

Original volume = $\pi D^3/6$ = 904778 mm^3

Change in volume = -904778 x 324.87 x 10^{-6} = -294 mm^3

WORKED EXAMPLE No. 5

In example No.3 the internal pressure is created by pumping water into the cylinder. Allowing for the compressibility of the water, deduce the volume of water at the outside pressure required to fill and pressurise the cylinder.

The bulk modulus K for water is 2.1 GPa.

SOLUTION

Initial volume of cylinder = V_1 = 13 253 600 mm^3 = volume of uncompressed water

Final volume of cylinder = V_2 = 13 257 600 mm^3 = volume of compressed water.

If V_2 was uncompressed it would have a larger volume V_3.

$V_3 = V_2 + \Delta V$ (all volumes refer to water).

From the relationship between pressure and volumetric strain we have

$\Delta V = pV_3/K = 0.8 \times 10^6 \times V_3/ 2.1 \times 10^9 = 380.9 \times 10^{-6}V_3$

$V_3 = 13\ 257\ 600 + 380.9 \times 10^{-6}V_3$

$0.9996V_3 = 13\ 257\ 600$

$V_3 = 13\ 262\ 700\ mm^3$

This is the volume required to fill and pressurise the cylinder. The answer is not precise because the mean dimensions of the cylinder were used not the inside dimensions.

SELF ASSESSMENT EXERCISE No. 2

1. A cylinder is 200 mm mean diameter and 1 m long with a wall 2.5 mm thick. It has an inside pressure 2 MPa greater than the outside pressure. Calculate the change in diameter and change in volume.
   Take E = 180 GPa and ν = 0.3
   (Answers 0.075 mm and 26 529 mm^3)

2. A sphere is 50 mm mean diameter with a wall 0.5 mm thick. It has an inside pressure 0.5 MPa greater than the outside pressure. Calculate the change in diameter and change in volume.
   Take E = 212 GPa and ν= 0.25
   (Answers 0.0022 mm and 8.68mm^3)

3a. A thin walled cylinder of mean diameter D and length L has a wall thickness of t. It is subjected to an internal pressure of p. Show that the change in length ΔL and change in diameter ΔD are

$\Delta L=(pDL/4tE)(1-2\nu)$ and $\Delta D=(pD^2/4tE)(2-\nu)$

b. A steel cylinder 2 m long and 0.5 m mean diameter has a wall 8 mm thick. It is filled and pressurised with water to a pressure of 3 MPa gauge. The outside is atmosphere.
For steel E= 210 GPa and ν=0.3. For water K = 2.9 GPa.

Calculate the following.
i. The maximum stress. (93.75 MPa)
ii. The increase in volume of the cylinder. (333092 mm^3)
iii. The volume of water at atmospheric pressure required. (392 625 000mm^3)

4a. A thin walled sphere of mean diameter D has a wall thickness of t. It is subjected to an internal pressure of p. Show that the change in volume ΔV and change in diameter ΔD are

$\Delta V=(3pDV/4tE)(1-\nu)$ where V is the initial volume.

b. A steel sphere 2m mean diameter has a wall 20 mm thick. It is filled and pressurised with water so that the stress in steel 200 MPa. The outside is atmosphere. For steel E= 206 GPa and ν = 0.3. For water K = 2.1 GPa.

Calculate the following.

i. The gauge pressure (8 MPa)

ii. The volume water required. (4.213 x 10^9 mm^3)