On completion of this tutorial you should be able to do the following.

- Explain inductance and inductors.
- Explain magnetic field and magnetic flux.
- Explain the motor principle.
- Explain the generator principle.
- Explain and describe an inductor.
- Define the unit of inductance.
- Define and calculate the energy stored in an inductor.

It is assumed that students already know the graphic symbols for electrical components and the basic theory of electricity, resistance, current, voltage (potential) and charge.
INTRODUCTION

An inductor is another example of a reactive component and its affect on a circuit is very dependant on the rate of change of current with time.

Students who are not familiar with basic electrical science will find other tutorials on the website covering the prerequisite material.

1. **REVISION OF BASIC ELECTRO-MAGNETISM.**

**MAGNETIC FLUX - \( \phi \) and FLUX DENSITY - \( B \)**

Magnetism produces lines of force known as flux. These can be detected by a compass needle and the pattern around a bar magnet is as shown. Flux is measured in Weber (Wb).

The flux \( \phi \) flows through the core of the magnet with the cross sectional area \( A \). The flux density is defined as

\[
B = \frac{\phi}{A}
\]

The units are Tesla (T) \( 1 \text{T} = 1 \text{Wb/m}^2 \)

The concept of flux density can be applied to any flux flowing through an area \( A \).

It is found that flux in an electro-magnet is directly proportional to the current \( i \), the number of turns \( n \) and inversely proportional to the length of the magnetic path \( l \). This means that \( i \, n / l \) is a constant for a given electro magnet and this is called the Magnetising force \( H \)

\[
H = \frac{i \, n}{l} \quad \text{(Ampere turns per metre)}
\]

The length of the magnetic circuit is often difficult to see because often the flux flows through the surrounding air as well as the core. If the coil is wound on a toroid the flux is entirely within the core and the length is the circumference.

When the coil is wound on a core with no magnetic properties the ratio of \( B/H \) has a constant value called the absolute permeability of free space symbol \( \mu_0 \)

\[
B/H = \mu_0 = 12.5 \times 10^{-7}
\]

If the material has any magnetic properties (usually iron based but other materials also) the ratio is dramatically changed by and the relative permeability \( \mu_r \) is introduced to correct the values.

\[
B/H = \mu_0 \, \mu_r
\]
2. **MOTOR PRINCIPLE**

Consider a conductor placed in a gap between the poles of a magnet. When current passes through the conductor, the circular lines of magnetism around it react with the straight lines from the magnet and produce a force on the conductor.

![Diagram of magnetic lines and force on a conductor](image)

The lines of magnetism between the north and south poles would rather pass over the top of the conductor because both lines are in the same direction on top. The lines behave like elastic bands and force the conductor down. If the direction of either the current or the magnetic field is reversed, the force will act up.

The force on the conductor is directly proportional to the current \( I \) (Amperes), the magnetic flux density \( B \) (Tesla) and the length of the conductor within the flux \( l \) (metres). It follows that

\[
F = BLI
\]

This is the important equation for the force on a conductor.

### SELF ASSESSMENT EXERCISE No. 3

1. The diagram shows a conductor 60 mm long carrying 12 Amperes in a flux of density 120 Tesla. Calculate the force acting on it and determine the direction of the force. (86.4 N left to right)

![Diagram showing force direction](image)

2. The diagram shows a conductor 80 mm long carrying 5 Amperes in a flux of density 20 Tesla. Calculate the force acting on it and the direction in which it moves. (8N Right to left)

![Diagram showing force direction](image)
3. **THE GENERATOR PRINCIPLE**

When a conductor is made to pass through a magnetic field, a voltage is generated in it. This voltage will be reduced slightly by the resistance of the conductor so we talk about the theoretical voltage as though the conductor had no resistance and this is called the ELECTRO MOTIVE FORCE or EMF with a symbol ‘E’ for a constant value and ‘e’ for an instantaneous value (when it is changing with time).

The EMF is directly proportional to the flux density B, the velocity v and the length of conductor within the flux l. The following is known as the generator equation.

\[ e = B l v \]

The direction of the current generated is found from Fleming’s Right Hand Rule. Point the index finger of your right hand in the direction of the flux (North to South). Point your thumb in the direction of the velocity. Bend over the second finger and it points the direction of the current.

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**SELF ASSESSMENT EXERCISE No. 4**

1. The diagram shows a conductor moving through a flux. The length is 80 mm and the flux density is 2 Tesla. The velocity is 12 m/s. Calculate the emf produced and the direction of the current.
   
   (1.92 V into the page)

2. The diagram shows a conductor moving through a flux. The length is 100 mm and the flux density is 1.8 Tesla. The velocity is 2 m/s. Calculate the emf produced and the direction of the current.
   
   (0.36 V out of page)
4. **INDUCTORS**

Any coil of wire is an inductor. A coil made specifically for an electric circuit is called an inductor. In this section we shall how the property of an inductor is used to influence the flow of current.

Inductors react to alternating current because of the electro-magnetic fields generated. They are basically a coil of conducting material wound on a former and they may be very small or very large.

Consider a coil in which alternating current is flowing. You should know that the current produces a magnetic field around the coil. If the current varies, then the magnetic flux varies and so the flux grows, then shrinks and then grows in the opposite sense. Since the lines of flux are cutting the turns on the coil, they must generate a back emf in the coil. This emf opposes the flow of the applied current (Lenz’s Law).

Faraday’s law gives the back emf: \[ e = - n \frac{d\phi}{dt} \]

An equal and opposite voltage must be applied to make the current flow hence there appears to be a resistance to the flow of current but this is not resistance it is called **REACTANCE**.

The flux is directly proportional to current. \[ \phi = k i \]

The rate of change of flux is hence directly proportional to the rate of change of current. \[ \frac{d\phi}{dt} = k \frac{di}{dt} \]

The back emf is hence \[ e = - nk \frac{di}{dt} \]

For a given inductor ‘nk’ is a constant called the **INDUCTANCE** with the symbol \( L \).

\[ k = \frac{L}{n} \]

\[ e = - L \frac{di}{dt} \]

The units of inductance are Henries (H). The base unit is very large and mH or \( \mu \text{H} \) are more common.

In the last section we had

\[ \frac{B}{H} = \mu_0 \mu_r \]

\[ B = \text{Flux density} = \phi/A \]

\[ H = \text{Magnetising Force} = i \frac{n}{l} \]

\( n \) is the number of turns again and \( l \) is the length of the magnetic circuit.

\[ \frac{B}{H} = \mu_0 \mu_r = 1 \frac{\phi(Ai n)}{(\phi/i)} (l/An) \]

\[ (\phi/i) = k \]

\[ \mu_0 \mu_r = k \frac{l}{An} \]

\[ k = \frac{L}{n} \]

\[ \mu_0 \mu_r = \frac{L}{l} \frac{1}{An^2} \]

\[ L = \frac{\mu_0 \mu_r An^2}{l} \]

This is a theoretical formula for the inductance of a coil.
WORKED EXAMPLE No. 1

A coil is 250 mm long and 25 mm diameter with 1500 turns on it. The core material has a relative permeability of 420.

Determine the inductance given $\mu_0 = 12.5 \times 10^{-7}$.

A current is made to vary in the coil at 300 A/s. Calculate the back emf.

SOLUTION

$$L = \frac{\mu_0 \mu_r A n^2}{l}$$

$$A = \pi \times 0.025^2/4 = 490.87 \times 10^{-6} \text{ m}^2.$$  
$$l = 0.25 \text{ m}$$

$$L = 12.5 \times 10^{-7} \times 420 \times 90.87 \times 10^{-6} \times 1500^2/0.25 = 2.33 \text{ Henries}$$

$$e = -L \frac{di}{dt} = -2.33 \times 300 = -700 \text{ Volts}$$

5. ENERGY STORED IN AN INDUCTOR

Suppose the current in an inductor is increased uniformly from 0 to $i$ Amps in time $t$ seconds. The rate of change of current is constant so $\frac{di}{dt} = \frac{i}{t}$ and the emf required is $L \frac{di}{dt} = Li/t$

The power grows from zero to $(V i)$ in time $t$ so the energy stored is the mean power x time

$$E = V i \frac{t}{2}$$  Substitute $V = Li/t$ and the energy stored is $E = L i^2/2$

SELF ASSESSMENT EXERCISE No. 5

1. An inductor is wound on a toroid 50 mm mean diameter and a round cross section 10 mm diameter. There are 300 turns and the relative permeability is 500. Calculate the inductance. (28 mH)

2. Calculate the energy stored in an inductor of value 1.7 H when 5 Amps flow. (21.25 J)

3. An inductor stores 5 Joules when 2 A flow in it. Calculate the inductance value. (2.5 H)