EDEXCEL NATIONAL CERTIFICATE/DIPLOMA

ADVANCED MECHANICAL PRINCIPLES AND APPLICATIONS
UNIT 18
NQF LEVEL 3

OUTCOME 2
BE ABLE TO DETERMINE THE STRESS DUE TO BENDING IN BEAMS AND TORSION IN POWER TRANSMISSION SHAFTS

TUTORIAL 1 - MOMENTS OF AREA

Direct stress due to bending: expressions for second moment of area of solid and hollow rectangular and circular beam sections; application of bending equation ($\sigma/y = M/I = E/R$) to determine stress due to bending and radius of curvature at a beam section; determination of factor of safety in operation

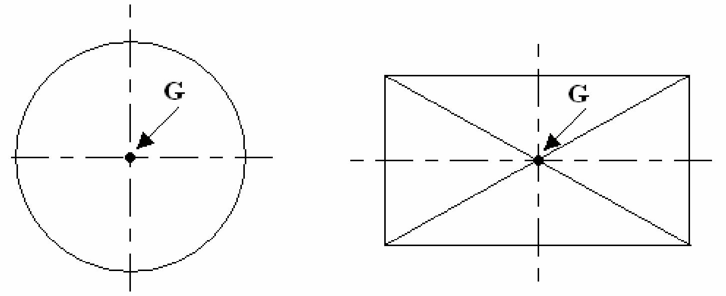
Shear stress due to torsion: expressions for polar second moment of area of solid and hollow circular transmission shaft sections; application of torsion equation ($\tau/r = T/J = G\theta/l$) and expression for power transmitted ($Power = T\omega$) to determine induced shear stress and angle of twist; determination of factor of safety in operation

This tutorial is not strictly necessary for students studying this unit unless you want to have a deeper understanding of second moments of area. Otherwise move on to tutorial 2.

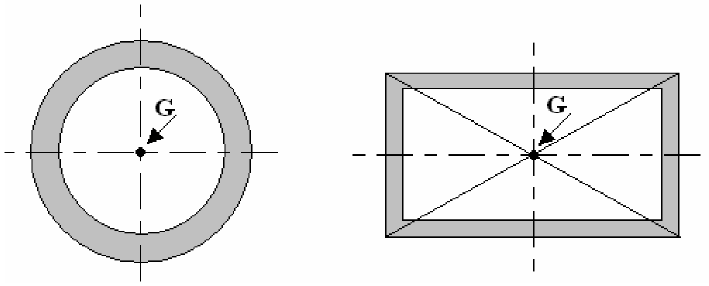
1. CENTROIDS and FIRST MOMENT OF AREA

In this unit you will learn how to calculate the stress due to bending in a beam with a rectangular or circular cross section. In order to find the centroid we must think of the cross sectional area as a thin plate. The centroid is the point where you could balance the shape on a pointed tip. It is the same as the centre of gravity for a real thin plate. The point is denoted with a letter G.

For rectangles and circles the centroid is simply the centre.

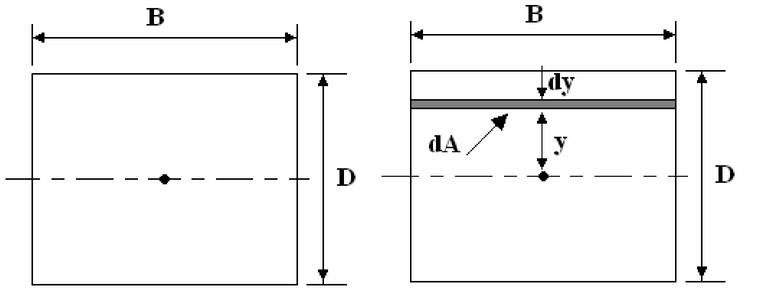


You also have to deal with hollow sections and so long as the wall is the same thickness every where the centroid remains in the same spot.



The first moment of area is found by multiplying an area by a distance. The distance is the distance from an axis about which may think it is revolving. In this unit we only consider the area as trying to revolve about an axis through the centroid and in the case of a rectangle parallel with an edge.

Consider a rectangle dimensions B and D. Consider a thin elementary strip as shown width B and height dy at distance y from the centroid. The area of the strip is dA = B dy. The moment of area is found by multiplying the area by y.



First moment of area of strip = B y dy

For the whole area we integrate between +D/2 and -D/2

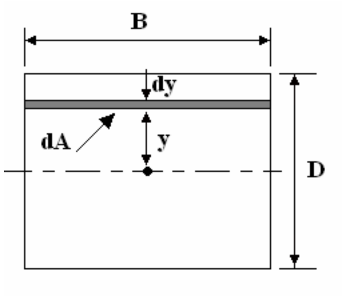
$$\text{First moment of area} = \int_{-D/2}^{+D/2} B\,y\,dy = B\left[\frac{y^2}{2}\right]_{-D/2}^{+D/2} = \frac{B}{2}\left\{\left[\frac{D^2}{4}\right] - \left[\frac{(-D)^2}{4}\right]\right\} = 0$$

We find that the first moment of area for the rectangle about the centroid is zero. This is true of the circle as well because the area is half above and half below. If you had to solve the first moment about the edge or any other axis the result would be different but you don't have to so we can go on to second moments of area.

2. SECOND MOMENTS OF AREAS

This could be a complicated piece of work to study but it is made a lot simpler by only having to deal with rectangles and circle. When we study how to find the stress in beams due to bending we will find that we need to calculate the second moment of area so it is an important concept. We usually denote this quantity with the letter I.

Consider the simple rectangle again. The second moment of area of the elementary strip about the axis through the centroid is:



$dI = dA\, y^2 = B\, y^2\, dy$

NOTE we multiply by the distance twice to get y^2 and this is why it is called a second moment.

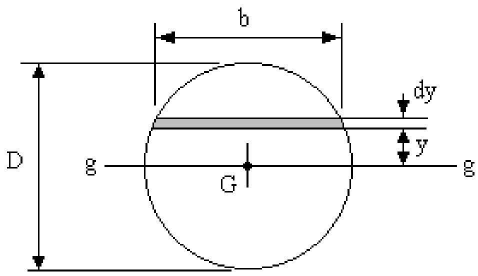
We integrate as before to find I

$$I = \int_{-\frac{D}{2}}^{\frac{D}{2}} By^2 dy = B\int_{-\frac{D}{2}}^{\frac{D}{2}} y^2 dy = B\left[\frac{y^3}{3}\right]_{-\frac{D}{2}}^{\frac{D}{2}} = \frac{B}{3}\left\{\left[\frac{D^3}{8}\right] - \left[\frac{(-D)^3}{8}\right]\right\} = \frac{BD^3}{12}$$

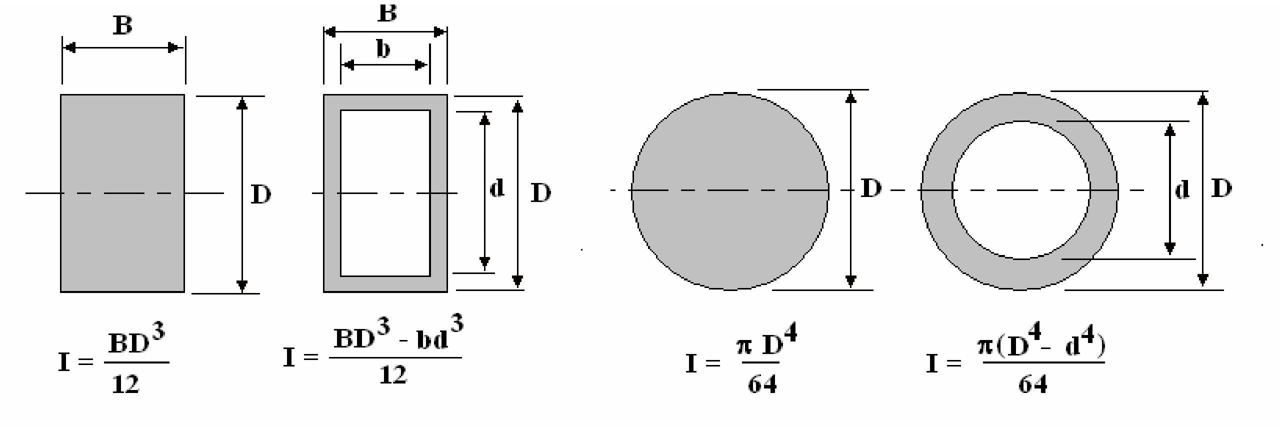
Because the distance is squared, the second moment of area of the half above the centre line adds to the half below and we do not get zero as for the first moment.

CIRCLES

The integration involved for a circle is complicated because the width of the strip b varies with distance y.



For the solid and hollow sections to be used in this unit the second moments of areas may be summarised as shown. These formulae should be remembered.



WORKED EXAMPLE No. 1

A beam has a rectangular cross section 80 mm wide and 120 mm deep. Find the second moment of area about the axis through the middle parallel to the short edge.

SOLUTION

B = 80 mm, D = 100 mm. It follows that the value of y that gives the maximum stress is 50 mm. Remember all quantities must be changed to metres in the final calculation.

$$I = \frac{BD^3}{12} = \frac{80 \text{ x } 100^3}{12} = 6.667 \text{ x } 10^6 \text{mm}^4 = 6.667 \text{ x } 10^{-6} \text{m}^4$$

WORKED EXAMPLE No.2

A beam has a hollow circular cross section 40 mm outer diameter and 30 mm inner diameter. Calculate the second moment of area about a diameter.
Calculate the following.

SOLUTION

D = 40 mm, d = 30 mm

I = $\pi(40^4 - 30^4)/64 = 85.9 \text{ x } 10^3$ mm^4 or $85.9 \text{ x } 10^{-9}$ m^4.

SELF ASSESSMENT EXERCISE No.1

1. Calculate the 2nd moment of area for a hollow round shaft about a diameter. It is 60 mm outside diameter and 40 mm inside diameter. ($510 \text{ x } 10^3$ mm^4)

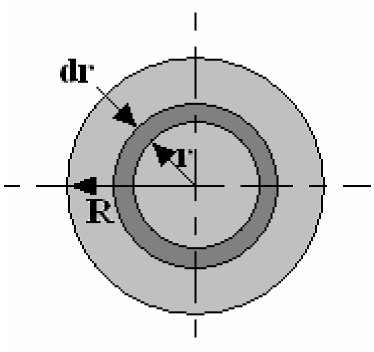
2. Calculate the 2nd moment of area for a hollow rectangular beam about an axis though the centroid parallel to the long edge. It is 60 mm wide and 40 mm deep on the outside with a wall thickness of 5 mm. ($207.5 \text{ x } 10^3$ mm^4)

3. Find the 2nd moment of area for a hollow round beam about its diameter. The outer diameter is 100 mm and the inner diameter is 60 mm. ($4.272 \text{ x } 10^6$ mm^4).

4. Find the 2nd moment of area of a hollow box section 80 mm wide and 80 mm deep with a wall thickness of 5 mm about the centre line. ($1.412 \text{ x } 10^6$ mm^4).

3 POLAR SECOND MOMENTS OF AREA

This is used in torsion of shafts. Consider a circular sectional area of radius R. Consider a small elementary ring of radius r and radial width dr.



The elementary ring may be seen as a thin rectangle of length 2πr and width dr. The area of the ring is then dA = 2π r dr. This becomes accurate as dr tends to zero.

The polar second moment of area is denoted J. The elementary ring has a small part of this dJ and this is found by multiplying dA by r^2.

$dJ = dA\ r^2 = 2\pi\ r^3\ dr$

For the whole section $J = 2\pi\int_0^R r^3 dr = 2\pi\left[\frac{r^4}{4}\right]_0^R = 2\pi\left\{\left[\frac{R^4}{4} - 0\right]\right\} = \frac{\pi R^4}{2}$

In terms of diameter R = R/2 $\quad J = \frac{\pi D^4}{32}$

For a hollow section outer diameter D and inner diameter d $\quad J = \frac{\pi\left(D^4 - d^4\right)}{32}$

WORKED EXAMPLE No. 3

Calculate the polar second moment of area for a circle 50 mm

SOLUTION

D = 0.05 m $\quad J = \frac{\pi D^4}{32} = \frac{\pi \text{ x } 0.05^4}{32} = 613.59 \text{ x } 10^{-9}\ m^4$

WORKED EXAMPLE No. 4

Calculate the polar second moment of area for a hollow circular section 60 mm outer diameter and 30 mm inner diameter.

SOLUTION

D = 60 mm d = 30 mm

$J = \frac{\pi\left(D^4 - d^4\right)}{32} = \frac{\pi\left(60^4 - 30^4\right)}{32} = 1.193 \text{ x } 10^6\ mm^4 \text{ or } 1.193 \text{ x } 10^{-6}\ m^4$

SELF ASSESSMENT EXERCISE No. 2

1. A shaft is made of solid round bar 30 mm diameter. Calculate the polar second moment of area for the cross section. (79522 mm^4)

2. A shaft is made from tube 25 mm outer diameter and 20 mm inner diameter. (22642 mm^4).